

## ExtraReview: Limits, Continuity, First Principles

1. Find the value(s) of a, b such that  $f(x)$  as defined below will be differentiable at  $x=1$ .

$$f(x) = ax^3 + bx \text{ for } x > 1$$

$$f(x) = 3x + 2 \text{ for } x \leq 1$$

2. Evaluate the following limits algebraically:

$$\text{a) } \lim_{x \rightarrow 16} \frac{4 - \sqrt{x}}{x - 16} \quad \text{b) } \lim_{x \rightarrow 3} \frac{1 - \frac{9}{x^2}}{x - 3} \quad \text{c) } \lim_{x \rightarrow \infty} \frac{3x^3 - x}{-3x^3 + 10x + 1} \quad \text{d) } \lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2 - 4}}{6x + 2}$$

3. Find the following limits using a calculator:

$$\text{a) } \lim_{x \rightarrow +\infty} \sqrt{x^2 + 6x + 10} - \sqrt{x^2 - 4x} \quad (\text{try this without a calculator}) \quad \text{b) } \lim_{x \rightarrow 1} \frac{2^{1/x} - 2}{x - 1}$$

$$\text{c) } \lim_{x \rightarrow 0^-} \frac{2^{1/x} - 2}{x - 1} \quad \text{d) } \lim_{x \rightarrow 0^+} \frac{2^{1/x} - 2}{x - 1} \quad \text{e) } \lim_{x \rightarrow 0} \frac{2^{1/x} - 2}{x - 1} \quad \text{f) Is the function}$$

$$f(x) = \frac{2^{1/x} - 2}{x - 1} \text{ continuous at either } x=0 \text{ or at } x=1?$$

4. Draw a graph satisfying the following properties:

$$\lim_{x \rightarrow \infty} f(x) = 2, \lim_{x \rightarrow -\infty} f(x) = -2, \lim_{x \rightarrow 2^+} f(x) = 4, \lim_{x \rightarrow 2^-} f(x) = 2, \lim_{x \rightarrow 1} f(x) = 0,$$

but

$$f(1) = -1$$

5. Use l'Hopital's rule or the first principles definition of the derivative to find:

$$\text{a) } \lim_{x \rightarrow 32} \frac{5x^{\frac{1}{5}} - 10}{\sqrt[3]{x + 32} - 4} \quad \text{b) } \lim_{h \rightarrow 0} \frac{(1+h)^{100} - 1}{h}$$

6. a) In the interval  $[0,4]$ , find the value(s) of  $x$  such that the average slope equals the instantaneous slope for the function  $f(x) = 2\sqrt{x^2 + 9}$

- b) For the same function, find the value of the constant  $k$  such that in the interval  $[0,k]$ , the average slope equals the instantaneous slope at  $x=4$

7. Find the derivative of  $f(x) = \frac{3}{\sqrt{2x+1}}$  by first principles

## Answers

1.  $a=-1, b=6$

2. a)  $-1/8$ , b)  $2/3$  c)  $-1$  d)  $-1/2$

3. a)  $5$  b) about  $-1.39$  c)  $2$  d)  $-\infty$  e) does not exist f) no

4. lots of possibilities

5. a)  $3$  b)  $100$

6. a)  $x = \sqrt{3}$  b) exact answer is  $k = \frac{40}{3}$

7.  $f'(x) = \frac{-3}{\sqrt{(2x+1)^3}}$