Exponents and Logarithms Review

1. Solving exponential equations:

Solve :

$a) 8^{x} = 4^{-x-3}$

$b) 3^{x+1} + 9^{x} = 18$

$c) 3x^{\frac{2}{3}} = \frac{1}{3}$

2. Recall: Terminology of Logarithms

If $10^{x} = 100$ then of course, $x =$ ___.

However, it is equivalent to write the original statement as $x = \log_{10} 100$.

In general, we can define this as if $b^{x} = a$ then $x = \log_{b} a$

Key Properties of logarithms

$a) \log_{a} b + lo_{a} c =$ _____

$b) \log_{a} b - \log_{a} c =$ _____

$c) n \log_{a} b =$ _____

$d) \log_{a} b = \frac{\log_{c} b}{\log_{c} a}$

used to change the base when needed

$e)\ a^{\log_{a} b} =$ _____

Proofs:

3. Simplify the following using the properties listed above:

a) $\log(20)+\log(5)=$ b) $\log_7 3 - \log_7 21 =$ c) $\log_3 9^{17} =$

d) $\log_4(x^2-4) - 2\log_4(x+2)$ e) $\log_8 \sqrt{8x}$

=

4.Solve exponential and logarithmic equations:

$a) 2^x = 100$

$b) 2\log(x-1) - \log(x+8) = -1$

5. The value of the constant e:

You are given $1 and are given an basic rate of interest of 100% per annum (very generous). You may compound this 100% rate as frequently as you wish. Find the limit of how much money you could end up with after one year.

6.The use of $e^x, \ln x$ in equations:

$Solve:$

$a) e^{2x} = 40$

$b) 3\ln x + 1 = -2$

c) Find the intersection of the graphs of $\begin{matrix} y = e^x + 3 \\ y = e^{2x} + 1 \end{matrix}$ algebraically.

d) Solve for x: $\frac{1}{\ln x} - 6\ln x = 1$

e) State the domain, range of y= xlnx

Exponents and Logarithms Problems

1. Solve for x:

$a) 2^{-x+1} = 4(8^x)$

$b) e^{2x} = 20$

$c) e^{2x} = e^x + 2$

$d) \log x = 2$

$e) \ln x = 2$

$f) \ln(\ln x) = 2$

$g) 10^{\log 20} = x$

$h) \log(9x+1) - \log(x-1) = 1$

$i) \ln x + \ln x^2 = 6$

2. Find the points of intersection of the following algebraically. You may check with your calculator.

a)
$$y = 2^x + 4^x$$
$$y = 2^{x+1} - 4^{x+1}$$

b) y=lnx and y = 2-3lnx

3. Solve each of the following:

a) $4^x + 4^{x-\frac{1}{2}} = 45$

b) $3^{p+1} + 3^p = 70$

c) $\left(3^m\right)\left(5^{m+1}\right) = 12^{2m-1}$

4. Find any intersection points of the graphs of $y = 2\log_3(9x)$ and $y = \log_3(x+8) + 2$.

5. Solve: $\log_x 4 + \log_8 \sqrt{x} = \frac{4}{3}$

6. Solve for x in the interval [0, 2π]:

a) $\log_2 \sin x + \log_2 \cos x = \frac{-3}{2}$

b) $\log_2 \cot x - 2\log_4 \csc 2x = \log_2 \cos x$

7. Find the domain, range of $y = \dfrac{x}{\ln x}$

8. Consider the functions $f(x) = \ln\left(\sin^2 x\right)$.

a) Find the domain, range of $f(x)$

b) Let $g(x) = 2\ln(\sin x)$. Explain why $f(x) \neq g(x)$.

Answers

1. a) x =-1/4 b) $\dfrac{\ln 20}{2}$ c) ln2 d) 100 e) e^2 f) $e^{(e^2)}$ g) 20 h) 11 I) e^2

2. a) x= $\dfrac{\ln(1/5)}{\ln 2}$ or $\log_2\left(\dfrac{1}{5}\right), y = \dfrac{6}{25}$ b) x=$e^{1/2}$ and y = 1/2

3. a) 2.45 b) $\dfrac{\ln(17.5)}{\ln 3} \doteq 2.61$ c) 1.81 4. (1,4) 5. 4,64

6. a) $\dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{9\pi}{8}, \dfrac{11\pi}{8}$ b) $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$

7. Domain=$\{x \in \mathbb{R} : x > 0, x \neq 1\}$, Range= $\{y \in \mathbb{R} : y < 0 \text{ or } y \geq e\}$

8. Domain=$\{x \in \mathbb{R}, x \neq k\pi, k \in I\}$, Range=$\{y \in \mathbb{R} : y \leq 0\}$.

b) Different domain as $g(x)$ is only valid when $\sin x \geq 0$