

Find the area bounded by the x and y axes and the curve

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

Solution:

$$\sqrt{y} = \sqrt{a} - \sqrt{x}$$

$$y = (\sqrt{a} - \sqrt{x})^2$$

$$y = a - 2\sqrt{ax}^{1/2} + x$$

We also know that we are finding the area in the first quadrant, so to find the bounds of integration, we need to find the x intercept.

$$\text{Let } y = 0 \Rightarrow x = a$$

$$\text{Area} = \int_0^a a - 2\sqrt{ax}^{1/2} + x \, dx$$

$$= \left[ax - \frac{4}{3} \sqrt{ax}^{3/2} + \frac{1}{2} x^2 \right]_0^a = a^2 - \frac{4}{3} a^2 + \frac{1}{2} a^2 = \frac{1}{6} a^2$$